

T-S-T Dual Black Hole

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Abstract

The sequence of intertwined T-S-T duality transformations acting on the 4D static uncharged black hole leads to a new black hole background with horizon and singularity exchanged. It is shown that this space-time is extendible too. In particular we will see that a string moving into a black hole is dual to a string leaving a white hole. That offers the possibility that a test-string does not see the singularity.

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The sequence of intertwined T-S-T duality transformations was first proposed by I.Bakas [1]. He argued that if one performs T-S-T duality transformations on a pure gravitational background with one Killing symmetry, one obtains a new background which will always be pure gravitational as well. Moreover, it turns out that T-S-T can be considered as a $SL(2, \mathbf{R})$ transformation in the space of string background metrics. This $SL(2, \mathbf{R})$ coincides with the action of the Ehlers-Geroch $SL(2, \mathbf{R})$ symmetry group of vacuum Einstein spaces upon reduction from four to three dimensions [2] provided that there is at least one Killing symmetry.

It is known that an exact conformal field theory describing a black hole in 2D space-time is an $SL(2, \mathbf{R})/U(1)$ gauged WZW model [3]. T-duality acting on such a background is well understood and leads to stringy properties of the black hole [4,5]. In the case of a Euclidean space-time the so called semi-infinite cigar is transformed to an infinite funnel. For a Lorentzian space-time duality interchanges different regions in the Kruskal-Szekeres coordinates[3, 4], especially duality exchanges the horizon with the singularity and the asymptotically flat space-time, which corresponds to the Lorentzian cigar, with the space-time inside the black hole singularity. In particular it has been shown that the 2D black hole is self dual.

Dual geometries of 4D black holes were studied in [6,7,8,9]. In [6] the T-duality transformation was applied to time translations of the Schwarzschild metric. The dual metric defined a geometry with naked singularities at $r = 0$ and $r = 2M$ and remained a spherically symmetric solution of the string background equations, but was not a black hole. Further it was shown [6] that a dual geometry with respect to the $SO(3)$ symmetry, i.e. duality with respect to non-abelian isometries is neither spherically symmetric nor asymptotically flat.

In the sequel we will show that the sequence of intertwined T-S-T duality transformations acting on the 4D black hole solution produces a new spherically symmetric background that is asymptotically flat and has a singularity at $r = 2M$ and a horizon at $r = 0$. In particular we will show that T-S-T duality transformation acts on the 4D static black hole just like the T-duality on the 2D black hole. Before starting with T-S-T let us recall that the action of T-duality for a nontrivial field configuration involving the metric, dilaton and antisymmetric tensor which are independent of the time coordinate is given by (for a review of T-duality [10]):

$$\begin{aligned}\tilde{G}_{00} &= \frac{1}{G_{00}}, & \tilde{G}_{0a} &= \frac{B_{0a}}{G_{00}}, & \tilde{B}_{0a} &= \frac{G_{0a}}{G_{00}} \\ \tilde{G}_{ab} &= G_{ab} - \frac{G_{0a}G_{0b} - B_{0a}B_{0b}}{G_{00}}, & \tilde{B}_{ab} &= B_{ab} - \frac{G_{0a}B_{0b} - G_{0b}B_{0a}}{G_{00}} \\ \tilde{\phi} &= \phi - \frac{1}{2} \log G_{00}.\end{aligned}$$

where '0' denotes the time direction. In order for the dual geometries to give string vacua they have to satisfy the string background equations to lowest order in α' [10,11].

The 4D uncharged static black hole solution can be regarded as special case of a gravitational string background with zero dilaton ϕ and anti-symmetric tensor field $B_{\mu\nu}$.

The isometry group of the Schwarzschild metric is given by time translations together with the $SO(3)$ space rotations.

The metric can be given in the form :

$$ds^2 = - \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right)^{-1} dr^2 + r^2 (d\theta^2 + \sin^2\theta d\phi^2).$$

The metric is singular for $r = 0$ and $r = 2M$. One can show [12] that $r = 0$ is a real singularity but $r = 2M$ just a coordinate singularity, reflecting deficiency in the used coordinate system and therefore being removable. If one calculates the Riemann tensor scalar invariant one finds:

$$R_{\mu\nu\lambda\sigma} R^{\mu\nu\lambda\sigma} = \frac{10M^2}{r^6}.$$

The scalar is finite at $r = 2M$ and diverges for $r \rightarrow 0$.

T-duality:

If we perform a T-duality transformation with respect to the t-coordinate we get the dual metric:

$$ds_D^2 = -\frac{dt^2}{1 - \frac{2M}{r}} + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2$$

and $d\Omega^2 = d\theta^2 + \sin^2\theta d\phi^2$. The dilaton is given by :

$$\phi_D = -\frac{1}{2} \log \left(1 - \frac{2M}{r}\right).$$

Metric and dilaton solve the string-frame background equations. If we switch to the Einstein-frame then the metric takes the form:

$$\bar{ds}_D^2 = -dt^2 + dr^2 + (r^2 - 2Mr) d\Omega^2$$

This metric defines a geometry with naked singularities at $r = 0$ and $r = 2M$, as it was already pointed out [6]. We can verify this by computing the dual scalar curvature

$$\mathcal{R} = \frac{2M^2}{(2M - r)^2 r^2}.$$

It is instructiv to compute again the Riemann tensor scalar invariant which is given by

$$R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} = \frac{3M^4}{(r^2 - 2Mr)^4}.$$

A T-duality transformation of the 4D black hole gives us a spherically symmetric solution of the string background equations with two naked singularities which is not a black hole.

S-duality:

Now we perform a S-duality transformation [13]. We have $B_{\mu\nu} = 0 \rightarrow b = 0$ and S-duality reduces to

$$\phi \longrightarrow -\phi,$$

and we set $-\phi = \hat{\phi}$. The metric in the Einstein-frame remains fixed under S-duality and is given by

$$\hat{ds}_D^2 = e^{-2\hat{\phi}_D} ds_D^2.$$

If we go back to the string-frame, we will find the metric

$$ds_D^2 = \left(1 - \frac{2M}{r}\right) dt^2 + \left(1 - \frac{2M}{r}\right) dr^2 + (r - 2M)^2 d\Omega^2$$

which together with the dilaton

$$\hat{\phi} = \frac{1}{2} \log\left(1 - \frac{2M}{r}\right)$$

solves the equation of motion in the string-frame.

T-duality:

We perform finally a T-duality transformation with respect to the t-coordinate. The T-S-T dual metric to the former Schwarzschild metric is then given by:

$$ds_{T-S-T}^2 = - \left(1 - \frac{2M}{r}\right)^{-1} dt^2 + \left(1 - \frac{2M}{r}\right) dr^2 + (r - 2M)^2 d\Omega^2.$$

The dual dilaton vanishes:

$$\begin{aligned} \phi_{T-S-T} &= \hat{\phi} - \frac{1}{2} \log\left(1 - \frac{2M}{r}\right) \\ &= 0 \end{aligned}$$

It is easy to check that the equation of motion are satisfied by the T-S-T dual metric. The metric becomes singular at $r = 0$ and $r = 2M$ but a calculation of the Riemann scalar invariant suggests that $r = 0$ is not a real physical singularity, but rather one which is a result of a bad choice of coordinates like $r = 2M$ in the Schwarzschild solution. The invariant is given by:

$$R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} = \frac{10M^2}{(r - 2M)^6} \ .$$

The curvature invariant takes at $r = M$ the same value as the invariant coming from the Schwarzschild metric. Our new space-time is defined for $r < 0$. Similar to the case of the Schwarzschild solution we have to check if $r = 0$ is a null hypersurface dividing the manifold into two disconnected components:

$$\begin{aligned} \text{I} & : \quad -\infty < r < 0 \ , \\ \text{II} & : \quad 0 < r < 2M \ . \end{aligned}$$

Inside region II the coordinates t and r reverse their character (t -spacelike, r -timelike). To get a better understanding of our new solution we have to prove that our solution can be extended when r tends to 0. I will follow the maximal extension procedure for the known Schwarzschild solution [12]. We start with a congruence of ingoing radial null geodesics given by

$$t = r - 2M \ln|r| + c \ ,$$

and $t \rightarrow -t$ defines outgoing radial null geodesics. In the following we suppress the integration constant $c = 2M \ln|2M|$. Now we change to a new time coordinate in which the ingoing geodesics become straight lines

$$t^* = t + 2M \ln|r| \ .$$

If we differentiate this equation with respect to r and substituting it for dt in the line element ds_{T-S-T}^2 , we find a new line element that we call dual Eddington-Finkelstein line element

$$ds_{T-S-T}^2 = - \left(1 - \frac{2M}{r}\right)^{-1} (dt^*)^2 + \frac{4M}{r - 2M} dt^* dr + \frac{r - 4M}{r - 2M} dr^2 + (r - 2M)^2 d\Omega^2 \ .$$

This solution is regular for the whole range $0 < r < 2M$. Our transformation $t \rightarrow t^*$ extends the coordinate range from $-\infty < r < 0$ to $-\infty < r < 2M$. The time reversed solution for outgoing radial null geodesics can be obtained by introducing another time coordinate $t_* = t - 2M \ln|r|$. If we introduce an advanced and a retarded null coordinate

$$v = t^* - r \quad , \quad w = t_* + r \quad ,$$

the corresponding dual Eddington-Finkelstein metric becomes:

$$\begin{aligned} \text{advanced: } ds_{T-S-T}^2 &= - \left(1 - \frac{2M}{r}\right)^{-1} dv^2 - 2dvdr + (r - 2M)^2 d\Omega^2 \\ \text{retarded: } ds_{T-S-T}^2 &= - \left(1 - \frac{2M}{r}\right)^{-1} dw^2 + 2dwdr + (r - 2M)^2 d\Omega^2 \quad . \end{aligned}$$

If we compare with the original advanced and retarded solutions coming from the Schwarzschild line element [12] we will find the advanced (retarded) dual solution coincide at $r = M$ with the retarded (advanced) original solution, latter are given by:

$$\begin{aligned} \text{advanced: } ds^2 &= - \left(1 - \frac{2M}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2 \\ \text{retarded: } ds^2 &= - \left(1 - \frac{2M}{r}\right) dw^2 - 2dwdr + r^2 d\Omega^2 \quad . \end{aligned}$$

Now let us make both extensions simultaneously, then we will get ds_{T-S-T}^2 in the coordinates (v, w, θ, ϕ) :

$$ds_{T-S-T}^2 = - \left(1 - \frac{2M}{r}\right)^{-1} dvdw + (r - 2M)^2 d\Omega^2 \quad ,$$

where r is determined implicitly by $\frac{1}{2}(v - w) = 2M \ln|r| - r$. For constant θ and ϕ the corresponding two-space is conformally flat which can be seen by defining $x = \frac{1}{2}(v - w)$ and $t = \frac{1}{2}(v + w)$. This two-space will be invariant under $v \rightarrow v' = v'(v)$ and $w \rightarrow w' = w'(w)$. If we define v', w' as

$$v' = \exp\left(\frac{v + 2M}{4M}\right) \quad , \quad w' = \exp\left(\frac{-w + 2M}{4M}\right) \quad ,$$

and introduce $x' = \frac{1}{2}(v' - w')$ and $t' = \frac{1}{2}(v' + w')$ we find the maximal extended line element and call it dual Kruskal line element (where we have included now the integration constant $c = 2M \ln|2M|$):

$$ds_{T-S-T}^2 = \frac{32M^3}{2M - r} \exp\left(\frac{r - 2M}{2M}\right) (-dt'^2 + dx'^2) + (r(t', x') - 2M)^2 d\Omega^2 \quad .$$

The Kruskal diagram is given by:

$$(t')^2 - (x')^2 = \frac{r}{2M} \exp\left(\frac{-r + 2M}{2M}\right)$$

At this point it would be reasonable to consider the original Kruskal line element given by

$$ds^2 = \frac{32M^3}{r} \exp\left(\frac{-r}{2M}\right) (-dt'^2 + dx'^2) + r(t', x')^2 d\Omega^2 \quad .$$

Kruskal's choice of the functions v', w' was $v' = \exp(v/4M)$, $w' = -\exp(-w/4M)$. The corresponding diagram is given by:

$$(t')^2 - (x')^2 = -\left(\frac{r}{2M} - 1\right) \exp\left(\frac{r}{2M}\right)$$

We observe that we can map the original solution to the dual via:

$$r \rightarrow r - 2M, \quad M \rightarrow -M \quad .$$

We have found a new background which solves the background field equations, that is singular at $r = 2M$ and has a horizon at $r = 0$. In particular T-S-T interchanges two asymptotically flat space-times both of which have a maximal extension. T-S-T duality interchanges region I ($r > 2M$) with region V ($r < 0$) while region II ($0 < r < 2M$) is transformed to itself (analog IV, VI are interchanged and III is transformed to itself). Further, we can conclude from our analysis above that a test-string can not distinguish if he moves on a ingoing radial null geodesic into the black hole or if he leaves a white hole in the dual geometry. That offers the possibility that a test-string does not see the singularity. From $M \rightarrow -M$ follows that the dual mass measured by an observer in region I is negativ.

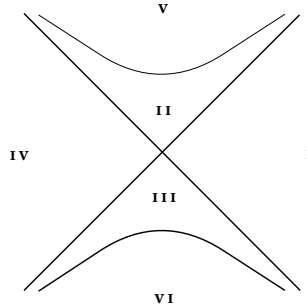


Figure 1: Extended space-time

All these facts are similar to the 2D case discussed in [4]. In the case of the 2D black hole one finds the positiv-mass black hole is dualized to a negative-mass solution, i.e. there is a mapping under duality of the asymptotically flat region to the region ‘beyond’ the singularity. Both regions are incorporated as different sectors of a single exact conformal field theory. Note that our discussed solutions are exact only in the leading

order of α' . Finally one has to keep in mind, although the physical equivalence of the dual solutions is expected to hold whenever the symmetry on which the dualization is based is compact, their equivalence for non-compact symmetries is not quite clear until now. About exactness in the compact and non-compact case see for instance [8,14,15].

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